1.2.0. FIRST ORDER DIFFERENTIAL EQUATIONS:

Most general form

$$\frac{dy}{dx} = f(x,y) = -\frac{P(x,y)}{Q(x,y)} \quad \ldots\ldots\ldots (2.1.0)$$

or $P(x,y)\,dx + Q(x,y)\,dy = 0$ .... (2.2.0)

A number of such equations exist in physical sciences.

special form

let $\frac{dy}{dx} = f(x,y) = -\frac{P(x)}{Q(y)}$ .... (2.3.0)

Then $P(x)dx + Q(y)\,dy = 0$

By integration $\int_{x_0}^{x} P(x)dx + \int_{y_0}^{y} P(y)dy = 0$ ..... (2.4.0)

We may ignore lower limits x_0 & y_0 which contribute to constants and simply add a constant.

Note! This separation of variable does not require that the differential eq^n be linear.

Ex: Boyle's law
In differential form $\frac{dV}{dP} = -\frac{V}{P}$ (T = constant)

or $\frac{dV}{V} = -\frac{dP}{P}$

Integrating $\ln V = -\ln P + C$

or $\ln PV = \ln k$ i.e. $PV = k$

Separable variable Equation:- .... (2.5.0)

$$\frac{dy}{dx} = f(x)g(y)$$

Here $\int \frac{dy}{g(y)} = \int f(x)\,dx$

If the integrals can be evaluated we get $y(x)$ satisfying (2.5.)

Note! Some ODE can be reduced to the form (2.5) after appropriate factorisation.

Ex. $\frac{dy}{dx} = x + xy = x(1+y)$

or $\int \frac{dy}{1+y} = \int x\,dx$

$\ln(1+y) = \frac{x^2}{2} + c \rightarrow$ Const.

$\therefore$

or $(1+y) = \exp\left(\frac{x^2}{2} + c\right) = A\exp\frac{x^2}{2}$ where $A = \exp(c)$

Solution Method

1. Factorise the eq^n so that it becomes separable
2. Rearrange terms involving x and y in opposite sides and integrate directly.
3. Constants of integration may be determined with further information (e.g. boundary cond.)

Exact Differential Equations :

Consider $P(x,y)\,dx + Q(x,y)\,dy = 0$ . . . . (2.1.0)

This eq^n is exact if it can be matched with differential $d\phi$

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy$$

As eqn. 2.1. has '0' on the right

$$\frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy = 0$$

i.e. $\phi(x,y) \Rightarrow$ The unknown f^n. = constant

Thus $P(x,y)\,dx + Q(x,y)\,dy = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy$

$$\therefore \left.\begin{aligned} P(x,y) &= \frac{\partial \phi}{\partial x} \\ Q(x,y) &= \frac{\partial \phi}{\partial y} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (2.6.0)$$

$$\therefore \frac{\partial P}{\partial y} = \frac{\partial^2 \phi}{\partial y \partial x} = \frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial Q}{\partial x}$$

$$\therefore \boxed{\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}} \quad \ldots\ldots\ldots\ldots (2.7.0)$$

This has some resemblance with equations involved in potential theory.

If $\phi(x,y)$ exists then our solution is

$$\phi(x,y) = c \quad \ldots\ldots\ldots\ldots (2.8.0)$$

From (2.6) $\phi(x,y) = \int P(x,y)\,dx + F(y)$ . . . . (2.9)

The function $F(y)$ can be found from (2.6) by differentiating (2.9) w.r.t y and equating to $Q(x,y)$

Ex. solve $x\frac{dy}{dx} + 3x + y = 0$

Rearranging $(3x+y)\,dx + x\,dy = 0$

Here $P(x,y) = 3x+y$ $\quad \therefore \frac{\partial P}{\partial y} = 1 = \frac{\partial Q}{\partial x}$

$Q(x,y) = x$

and the eqn is exact.

Hence the solution is given by (2.9)

$$\phi(x,y) = \int (3x+y)\,dx + F(y) = c_1$$

$$\text{or } \frac{3x^2}{2} + xy + F(y) = c_1$$

Now $\frac{\partial \phi}{\partial y} = \frac{\partial F(y)}{\partial y} + x = Q(x,y) = x$

Thus $\frac{\partial F(y)}{\partial y} = 0$ or $F(y) = C_2$

Thus $\phi(x,y) = \frac{3x^2}{2} + xy + C_2 = C_1$

or $\frac{3x^2}{2} + xy = C \Rightarrow$ solution of original ODE.

Method of Solution :- (i) check if the eq^n is exact using 2.7.
(ii) solve using 2.9
(iii) Find $F(y)$ by differentiating 2.9 w.r.t y and using 2.6.

1.2.1. INEXACT EQUATIONS :-

The equations may be written in the form

$$P(x,y)\,dx + Q(x,y)\,dy = 0 \quad \text{but} \quad \frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$$

and are called inexact equations.
Here the differential can be made exact by multiplying by a factor called integrating factor ($\mu(x,y)$ say)
which obeys $\frac{\partial}{\partial y}(\mu P) = \frac{\partial}{\partial x}(\mu Q)$ . . . . . . (2.1.1)

There exists no general methods for finding the integrating factor $\mu(x,y)$. Sometimes it may be found by inspection. However if an integrating factor exists which is a f^n of x or y alone then eqn. 2.1.1 can be solved to find it.

Let $\mu = \mu(x)$

Then $\mu(x)P(x,y)dx + \mu(x)Q(x,y)dy = 0$

and eqn. 2.1.1 reads as

$$\mu(x)\frac{\partial P}{\partial y}(x,y) = Q(x,y)\frac{\partial \mu(x)}{\partial x} + \mu(x)\frac{\partial Q}{\partial x}(x,y)$$

or $\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = \frac{Q}{\mu}\frac{d\mu}{dx}$

or $\frac{d\mu}{\mu} = \frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)dx = f\,dx$

or $\ln \mu = \int f\,dx$ . . . . (2.1.2)

If we insist that $f = f(x)$ only

then $\mu(x) = \exp\left[\int f(x)\,dx\right]$

with $f(x) = \frac{1}{Q}\left[\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right]$ . . . . (2.1.3)

similarly if $\mu = \mu(y)$

$\mu(y) = \exp\left[\int g(y)\,dy\right]$ . . . . (2.1.4)

where $g(y) = \frac{1}{P}\left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right]$ . . . . . . (2.1.5)

Ex: $\frac{dy}{dx} = -\frac{2}{y} - \frac{3y}{2x}$ . . . . (2.1.6)

Rearranging $\frac{dy}{dx} = \frac{-4x - 3y^2}{2xy}$

or $2xy\,dy = -4x\,dx - 3y^2 dx$

or $2xy\,dy + (3y^2+4x)\,dx = 0$ . . . . (2.1.7)

Here $Q = 2xy$; $P = 3y^2+4x$

and $\frac{\partial P}{\partial y} = 6y \neq \frac{\partial Q}{\partial x} (= 2y)$

Thus the ODE is inexact

However, we notice that

$$\frac{1}{Q}\left[\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right] = \frac{1}{2xy}[6y - 2y] = \frac{2}{x}$$

$\Rightarrow$ f^n of x alone $= f(x)$

Hence there exists an integrating factor which is a function of x alone given by

$$\mu(x) = \exp\left[\int f(x)dx\right] = \exp\left[\int \frac{2}{x} dx\right] = \exp[2\ln x]$$
$$= \exp[\ln x^2] = x^2$$

Multiplying eq^n 2.1.7 by x^2 we get

$x^2(3y^2+4x)\,dx + x^2(2xy)\,dy = 0$

or $(3x^2y^2 + 4x^3)\,dx + 2x^3y\,dy = 0$

or integrating we get

$x^4 + x^3y^2 = c$

or $x^4 + x^3y^2 = c$

$\downarrow$

$\Rightarrow$ soln of original eq^n $\leftarrow$

or

$\phi(x,y) = \int P(x,y)dx + F(y)$

$= \int (4x^3 + 3x^2y^2)dx + F(y)$

$= x^4 + x^3y^2 + F(y) + C_1$

$\frac{\partial Q}{\partial y} = 2x^3y + \frac{\partial F}{\partial y}$

$= Q(x,y) = 2x^3y$

$\therefore \frac{\partial F}{\partial y} = 0$

Hence $\phi(x,y) = x^4 + x^3y^2 + C$

Method of soln:

- Examine if $f(x)$ and $g(y)$ are f^ns of only x or y
- If so, then the integrating factor is a f^n of either x or y and is given by eqn 2.1.3 & 2.1.5
- If the integrating factor is a f^n of x & y, then it may sometimes be found by inspection or by trial and error.
- In any case the integrating factor must satisfy eq. 2.1.1.
- once the eq^n has been made exact. Solve by the method of section 2.1.

1.2.2. Linear First Order Differential Equations:-

consider the eq^n 2.1

$$\frac{dy}{dx} = f(x, y) \quad \ldots \quad 2.1$$

If $f(x,y) = -p(x)y + q(x)$ 2.2.1

then $\frac{dy}{dx} = -p(x)y + q(x)$

or $\frac{dy}{dx} + p(x)y = q(x)$ 2.2.2

This is the most general linear first order DE.

If $q(x) = 0$, eq. (2.2.2) is homogeneous.

$q(x) \neq 0$ gives a source term or a driving term.

Eq.(2.2.2) is linear, each term is linear in y or $\frac{dy}{dx}$. There are no higher powers i.e. y^2 and no products i.e. $y\frac{dy}{dx}$. Note that the linearity refers to independent variable y and $\frac{dy}{dx}$, $p(x)$ and $q(x)$ need not be linear in x. Eq (2.2.2), the most important of the first DE in physics may be solved exactly.

The equation canbe made exact by multiplying throughout by an appropriate integrating factor $\mu(x)$ such that

$$\mu(x)\frac{dy}{dx} + \mu(x)p(x)y = \mu(x)q(x) \quad \ldots \quad 2.2.3$$

may be re written as

$$\frac{d}{dx}[\mu(x)y] = \mu(x)q(x) \quad \ldots \quad 2.2.4$$

The purpose of this is to make the left side of eq (2.2.2) a derivative so that it can be integrated by inspection. It also incidentally, makes eq. 2.2.2 exact.

Expanding eq.(2.2.4) we get

$$\mu(x)\frac{dy}{dx} + \frac{d\mu}{dx}y = \mu(x)q(x)$$

comparison with eqn (2.2.3) shows that we must require

$$\mu(x)\frac{dy}{dx} + \frac{d\mu}{dx}y = \mu(x)\frac{dy}{dx} + \mu(x)p(x)y$$

i.e. $\frac{d\mu(x)}{dx} = \mu(x)p(x)$ 2.2.5

μ and x separable.

This is a D.E for $\mu(x)$ with μ and x separable.

$\frac{d\mu}{\mu} = p(x)dx$ or $\mu = e^{\int p(x)dx}$ 2.2.6

This is the integrating factor.

With this we proceed to integrate eq. 2.2.4 which is what we want

Here $\int \frac{d}{dx}[\mu(x)y]dx = \int \mu(x)q(x)dx$

or $\mu(x)y = \int \mu(x)q(x)dx + C$

$$\therefore\ y(x) = [\mu(x)]^{-1}\left\{\int^{x}\mu(x)\,q(x)\,dx + c\right\} \quad \ldots 2.2.7$$

with $\mu(x) = e^{\int p(x)dx}$ we have finally

$$y(x) = e^{-\int^{x} p(x)dx}\left[\int^{x} q(x')\,e^{\int^{x'} p(x')\,dx'}\,dx' + c\right] \quad \ldots 2.2.8$$

Eq 2.2.8 is the most general soln of eq^n 2.2.2. The soln

2.2.9 $y_1(x) = c\,e^{-\int^{x} p(x)dx}$ corresponds to the case $q(x) = 0$

and is a general solution of the homogeneous DE.

The other term in eq. 2.2.7

$$y_2(x) = \left[e^{-\int^{x} p(x)dx}\right]\int^{x} q(x')\,e^{\int^{x'} p(x')dx'}\,dx' \quad \ldots\ldots 2.2.10$$

is a particular solution corresponding to specific source term $q(x)$.

<u>Note</u>: Linear first order DE is separable if it is homogeneous [i.e. $q(x) = 0$]. otherwise, except for special cases like $p = \text{const}$; $q = \text{const}$ or $q(x) = \lambda p(x)$; eq (2.2.2) is not separable.

<u>Ex.</u> $\frac{dy}{dx} = 4x - 2xy$ Here $p(x) = 2x$ and integrating factor is $\mu(x) = e^{\int p(x)dx}$

or $\mu(x) = e^{\int 2x dx} = e^{x^2}$

Multiplying by x^2 the DE becomes $e^{x^2}\frac{dy}{dx} + 2xy\,e^{x^2} = 4x\,e^{x^2}$

or $\frac{d}{dx}\left[y e^{x^2}\right] = 4x e^{x^2} \Rightarrow y e^{x^2} = \int 4x e^{x^2} dx + c = 4\int e^{x^2} x\,dx + c$

$= 2\int e^{x^2} d(x^2) + c = 2e^{x^2} + c$

Thus the soln of the ODE is $y = 2 + c e^{-x^2}$

<u>Ex. LR circuit</u>: For a circuit containing resistance and inductance <u>Kirchhoff's Law</u> gives $L\frac{dI(t)}{dt} + RI(t) = V(t)$

$V(t)$ = Voltage

$I(t)$ = current at time t

or $\frac{dI(t)}{dt} + \frac{R}{L}I(t) = \frac{1}{L}V(t)$ The I.F is

$$\mu(t) = e^{\int \frac{R}{L}dt} = e^{\frac{Rt}{L}}$$

Hence by eq. 2.2.7

$$I(t) = e^{-\frac{Rt}{L}}\left[\int^{t} e^{\frac{Rt}{L}}\,\frac{V(t)}{L}\,dt + c\right]$$

c is to be determined from boundary cond.

For $V(t) = V_0 = \text{constant}$

$$I(t) = e^{-\frac{Rt}{L}}\left[\frac{V_0}{L}\int e^{\frac{Rt}{L}}dt + c\right] = e^{-\frac{Rt}{L}}\left[\frac{V_0}{L}\,\frac{L}{R}\,e^{\frac{Rt}{L}} + c\right]$$

$$= \frac{V_0}{R} + c e^{-\frac{Rt}{L}}$$

If initial condition gives $I(0) = 0$ then $c = -\frac{V_0}{R}$

and $I(t) = \frac{V_0}{R}\left[1 - e^{-\frac{R}{L}t}\right]$

1.3. HOMOGENEOUS EQUATIONS:

Homogeneous equations are ODE's written in the form

$$\frac{dy}{dx} = \frac{P(x,y)}{Q(x,y)} = F\left(\frac{y}{x}\right) \quad \ldots\ldots \quad 1.3.1.$$

where $P(x,y)$ & $Q(x,y)$ are homogeneous functions of identical degree.

A f^n $f(x,y)$ is homogeneous of degree n if for any λ, it obeys

$$f(\lambda x, \lambda y) = \lambda^n f(x,y)$$

$$\left.\begin{aligned} P(x,y) &= x^2y - xy^2 \\ Q(x,y) &= x^3 + y^3 \end{aligned}\right\} \text{ are homogeneous in } x \text{ \& } y \text{ and of degree 3.}$$

For $P(x,y)$ & $Q(x,y)$ both to be homogeneous and of same degree the sum of powers of x and y ~~shoulded~~ should be identical for each term in P and Q.

The RHS of a homogeneous f^n can be written as a f^n of y/x.

let $\frac{dy}{dx} = F(y/x)$

substitute $y = vx$

$$\therefore \frac{dy}{dx} = x\frac{dv}{dx} + v = F(v)$$

This is separable as the transformed eq^n is

$$x\frac{dv}{dx} + v = F(v)$$

$$\text{or} \quad x\frac{dv}{dx} = F(v) - v \Rightarrow \frac{dv}{F(v)-v} = \frac{dx}{x} \quad \ldots\ldots \quad 1.3.2$$

Direct integration gives the soln of the ODE depending on the form of $F(v)$

Ex. $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$

Put $y = vx$

$$\therefore v + x\frac{dv}{dx} = v + \tan v$$

$$\therefore \int \frac{dv}{\tan v} = \int \frac{dx}{x} \Rightarrow \int \cot v\, dv = \ln x + c_1$$

But $\int \cot v\, dv = \int \frac{\cos v\, dv}{\sin v} = \int \frac{d(\sin v)}{\sin v} = \ln(\sin v) + c_2$

Hence $\ln \sin v = \ln x + c_1 - c_2$

$\therefore \ln\left(\frac{\sin v}{x}\right) = \ln A$ say

or $\frac{\sin v}{x} = A$ const. $\Rightarrow v = \sin^{-1}(Ax)$

$\therefore y = vx = x\sin^{-1}(Ax) \Rightarrow$ soln of DE.

Method

- check if the eq^n is homogeneous
- If so, substitute $y = vx$, the separate variables as in 1.3.2. and integrate directly.
- Finally replace v by y/x to get the soln.

1.4. BERNOULLI'S EQUATION :-

$$\frac{dy}{dx} + P(x)y = Q(x)y^n \qquad n \neq 0 \text{ or } 1 \quad \ldots\ldots 1.4.1$$

For $n=0$ or 1 it is linear

The eq^n is non linear due to the term y^n

This eq^n can be made linear by the transformation

$$v = y^{1-n}$$

Then $\frac{dv}{dx} = (1-n)y^{-n}\frac{dy}{dx}$

$\therefore \frac{dy}{dx} = \frac{y^n}{1-n}\frac{dv}{dx}$

substitution in eq. (1.4.1) gives

$$\left(\frac{y^n}{1-n}\right)\frac{dv}{dx} + P(x)\frac{v}{y^{-n}} = Q(x)\frac{v}{y^{1-2n}}$$

or $\frac{dv}{dx} + (1-n)P(x)v = (1-n)Q(x)$

which is a linear equation and may be solved by choosing appropriate integrating factor.

Ex. $\frac{dy}{dx} + \frac{y}{x} = 2x^3y^4 \Rightarrow$ Let $v = y^{1-4} = y^{-3}$ then $\frac{dy}{dx} = \frac{dy}{dv}\cdot\frac{dv}{dx} = -\frac{y^4}{3}\frac{dv}{dx}$

$\therefore -\frac{y^4}{3}\frac{dv}{dx} + \frac{vy^4}{x} = 2x^3y^4 \Rightarrow \frac{dv}{dx} - \frac{3v}{x} = -6x^3$

Take the I.F $= \exp\left[-3\int\frac{dx}{x}\right] = \exp[-3\ln x] = \frac{1}{x^3}$

This yields $\frac{v}{x^3} = -6x + c \quad \therefore y^{-3} = -6x^4 + cx^3 \Rightarrow$ soln.

Note: There are many other types of first order DE which we just skip.

1.5. MISCELLANEOUS EQ^N.

Consider the type

$$\frac{dy}{dx} = F(ax+by+c) \quad \ldots\ldots \quad 1.5.1$$

$\Rightarrow$ solve by change of variable method

put $v = ax + by + c$

$$\frac{dv}{dx} = a + b\frac{dy}{dx}$$

Eq. 1.5.1 $\Rightarrow \frac{1}{b}\left(\frac{dv}{dx} - a\right) = F(v)$

or $\frac{dv}{dx} = bF(v) + a$

This is separable and may be integrated directly.

Ex.
$$\frac{dy}{dx} = (x+y+1)^2$$

put $v = x+y+1$

$\therefore \frac{dv}{dx} = \frac{dy}{dx} + 1 \Rightarrow \frac{dv}{dx} = v^2 + 1$ or $\frac{dv}{1+v^2} = dx$

$\therefore$ integrating we get

$$\tan^{-1}(v) = x + c_1$$

or $\tan^{-1}(x+y+1) = x + c_1$

The soln of DE is $\tan^{-1}(x+y+1) = x + c_1$

Ex. Flow of water from an orifice in a tank.

Here velocity $v = \sqrt{2gh}$

Volume of water ΔV escaping in Δt

$\Delta V = vA\Delta t$ A = Area of the hole

$= \sqrt{2gh}\,\frac{\pi d^2}{4}\Delta t$

Volume lost in water $\propto$ level of tank

$$\Delta V = -\frac{\pi D^2}{4}\Delta h$$

$$\therefore -\frac{\pi D^2}{4}\Delta h = \sqrt{2gh}\,\frac{\pi d^2}{4}\Delta t$$

or $\frac{dh}{dt} = -\sqrt{2gh}\,\frac{d^2}{D^2} = -\frac{d^2}{D^2}\sqrt{2g}\sqrt{h}$ or $\frac{dh}{h^{1/2}} = -\frac{d^2}{D^2}\sqrt{2g}\,.dt$

Integrating $2h^{1/2} = -\frac{d^2}{D^2}\sqrt{2g}\,.t + c$

or $h^{1/2} = -\frac{d^2}{D^2}\sqrt{g/2}\,t + c$

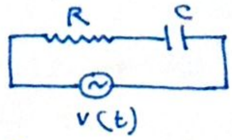
Boundary cond. $h = h_0$ at $t = 0$

$\therefore c = h_0^{1/2}$

Hence $h(t) = \left[-\frac{d^2}{D^2}\sqrt{\frac{g}{2}}\,t + \sqrt{h_0}\right]^2$

$h(t) = 0$ at $t = \frac{D^2}{d^2}\sqrt{\frac{2h_0}{g}}$ sec.

Ex. In a resistance capacitor series circuit the charge q on the capacitor is given by



$$R\frac{dq}{dt} + \frac{q}{c} = V(t)$$

Boundary condition at $t=0$, $q=0$

and $V(t) = V_0 \sin\omega t$

Here $\frac{dq}{dt} + \frac{1}{RC} q = \frac{V(t)}{R}$

Here I.F is $\exp\left[\int \frac{1}{RC} dt\right] = e^{t/RC}$

Thus the soln is given by $q(t) = e^{-t/RC}\left[\int^t \frac{V(t)}{R} e^{t/RC} dt + c_1\right]$

If $V(t) = \text{Const} = V_0$ then

$$q(t) = e^{-t/RC}\left[\frac{V_0}{R}\int e^{t/RC} dt + c_1\right] = e^{-t/RC}\left[\frac{V_0}{R} \cdot RC e^{\frac{t}{RC}} + c_1\right]$$

$$= q_0 + c_1 e^{-t/RC}$$

At $t=0$, $q=0$ $\therefore$ $c_1 = -q_0$

$\therefore$ $q(t) = q_0\left[1 - e^{-t/RC}\right]$ $\Rightarrow$ condenser charging

If $V(t) = V_0 \sin\omega t$

then $q(t) = e^{-t/RC}\left[\int \frac{V(t)}{R} e^{t/RC} dt + c_1\right]$

If $V(t) = V_0 \sin\omega t$

$$\int \frac{V(t)}{R} e^{t/RC} dt = \left(\frac{V_0}{R}\right)\int \sin\omega t\, e^{t/RC} dt = \frac{V_0}{R} I$$

where $I = \int \sin\omega t\, e^{t/RC} dt$

$$I = \left[RC \sin\omega t\, e^{t/RC} - \omega RC \int \cos\omega t\, e^{t/RC} dt\right]$$

$$= \left[RC \sin\omega t\, e^{t/RC} - \omega RC\left\{RC\cos\omega t\, e^{t/RC} + \omega RC \int \sin\omega t\, e^{t/RC} dt\right\}\right]$$

$$= RC \sin\omega t\, e^{t/RC} - \omega R^2c^2 \cos\omega t\, e^{t/RC} - \omega^2 R^2 c^2 I$$

Thus $(1+\omega^2R^2c^2) I = \{RC\sin\omega t - \omega R^2c^2 \cos\omega t\} e^{t/RC}$

$\therefore$ $I = (1+\omega^2R^2c^2)^{-1}\{RC\sin\omega t - \omega R^2c^2\cos\omega t\} e^{t/RC}$

Hence $q(t) = e^{-t/RC}\left[\frac{V_0}{R}(1+\omega^2R^2c^2)^{-1} RC\{\sin\omega t - \omega RC\cos\omega t\} \times e^{t/RC} + c_1\right]$

$$= q_0 (1+\omega^2R^2c^2)^{-1}(\sin\omega t - \omega RC\cos\omega t) + c_1 e^{-t/RC}$$

Applying boundary condition viz at $t=0$, $q=0$. $c_1 = q_0(1+\omega^2R^2c^2)^{-1}\omega RC$

Thus $q(t) = \left[q_0(1+\omega^2R^2c^2)^{-1}\{\sin\omega t - \omega RC\cos\omega t\} + q_0(1+\omega^2R^2c^2)^{-1}\omega RC\, e^{-t/RC}\right]$

$$= q_0(1+\omega^2R^2c^2)^{-1}\left[\sin\omega t + \omega RC\{e^{-t/RC} - \cos\omega t\}\right]$$

1.6. Most general higher degree first order eqn :-

$F(x, y, \frac{dy}{dx}) = 0$ 1.6.1

The most general standard form is

$p^n + a_{n-1}(x,y)p^{n-1} + \cdots\ a_1(x,y)p + a_0(x,y) = 0$ . . . 1.6.2

where $p = \frac{dy}{dx}$

we shall not discuss the solutions of such equations (if it can be solved).